Enrollment No.................

Master of Science (Mathematics)
Fourth Semester Main Examination, Aug-Sep 2020
Functional Analysis-II [MSM401T]

Time: 3:00 Hrs **Max Marks 85**

Note : Attempt all questions. Question no. 1 to Q. no.5 has two parts. Part A is 10 marks and part B is 7 marks.

Q.1 (a) Show that the norm of an isometry is one.
(b) Explain Bessel inequality.

OR

(a) Explain Inner Product Space.
(b) Define orthonormal sets.

Q.2 (a) Define total orthonormal. Let μ be a subset of an inner product space X, which is total in X. then prove that $x \perp M \Rightarrow \ x = 0$
(b) State and prove Riesz's theorem.

OR

(a) State and prove Riesz representation theorem.
(b) Let H be a separable Hilbert space then prove that every orthonormal set in H is countable.

Q.3 (a) Explain Hilbert adjoint operators.
(b) If H is complex and $< T_x, x >$ is real for all $x \in H$ the operator T is self-adjoint. Prove it T:H $\Rightarrow$ H be a bonded linear operator on a Hilbert space H.

OR

(a) Explain normal operator.
(b) Show that the unitary operators on a Hilbert space H from a group.

Q.4 (a) State and Prove Uniform Boundedness theorem.
(b) Explain weak convergence.

OR

(a) Explain strong convergence.
(b) If a normed space X is reflexive, show that X' is reflexive.

Q.5 (a) State and Prove Closed graph theorem.
(b) Show that an open mapping need not map closed sets onto closed sets.

OR

(a) Prove that a bounded linear operator. T from a Banach space X onto a Banach space y has the property that the image $T(B_o)$ of the open unit ball Bo= B(0,1) ⊂ X contains an open ball about $0 \in y$.
(b) Define closed linear operators..

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Master of Science (Mathematics)
Fourth Semester Main Examination, Aug-Sep 2020
Advanced Special Function-II [MSM402T]

Time: 3:00 Hrs **Max Marks 85**

Note: Attempt all questions. Each question has two parts.
Part A is 10 marks and part B is 7 marks.

Q.1 (a) Derive the Rodrigue's formula.

$$P_n^{'}(x) = \frac{1}{2^n n!} \frac{a^m}{dx^n} (x^2 - 1)^n$$

(b) Prove that: $2xH_n(x) = 2nH_{n-1}(x) + H_{n+1}(x)$

OR

(b) Show that

$$(n+1)L_{n+1}(x) = (2_{n+1} - x)L_n(x) - nL_{n-1}(x)$$

(c) Prove Rodriguez's formula for Hermite polynomial.

Q.2 (a) Define Laguerre Polynomial Prove that

$$L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n} (x^n . e^{-x})$$

(b) Write any two properties of Laguerre polynomial.

OR

(a) Prove that

$$D\{L_n{}^{(\propto)}(x)\} = D\{L_{n-1}{}^{(\propto)}(x)\} - L_{n-2}{}^{(\propto)}(x)$$

(b) Find the value of $L_n(x)$ for n=0, 1, 2, 3 and 4.

Q.3 (a) Prove that

$$\sum_{n=0}^{\infty} t^n L_n{}^{(\propto)}(x) = \frac{1}{(1-t)^{\propto+1}}^{\frac{-t^x}{1-t}}$$

(b) Define generalized Laguerre polynomial.

OR

(a) Show that

(i) $L_n(0) = -n$

(ii) $L_n^{"}(0) = \frac{\{n(n-1)\}}{2}$

(a) Prove that

$$\int_x^{\infty} e^{-y} L_n(y) dy = e^{-x}[L_n(x) - L_{n-1}(x)]$$

Q.4 (a) Show that

$$T_n(-x) = (-1)^n T_n(x)$$

(b) Give trigonometric definite of chebyshow polynomial.

OR

(a) Show that $\frac{1}{\sqrt{1-x^2}} U_n(x)$ satisfies the differential equation.

$$(1-x^2)\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+(n^2-1)u=0$$

(b) Define $T_n(x)$ and show that

$$\sum_{n-0}^{\infty}T_n(x)=\frac{1-xt}{1-2xt+t^2}$$

Q.5 (a) Show that

$P_n^{(\propto,\beta)}(\text{-z})=(-1)^n\,P_n^{(\beta,\propto)}(z)$

(b) Prove orthogonelity for Jacobi polynomial.

OR

(a) Obtain Batesman's generating function for $P_n(x)$.

(b) Prove that

$2_nJ_n(z)=z[J_{n-1}(z)+J_{n+1}(z)]$

Enrollment No.................

Master of Science (Mathematics)

Fourth Semester Main Examination, Aug-Sep 2020

Theory of linear Operator-II [MSM403T]

Time: 3:00 Hrs **Max Marks 85**

Note : Attempt all questions. Question no. 1 to Q. no.5 has two parts. Part A is 10 marks and part B is 7 marks.

Q.1 (a) State and prove fredholum alternative theorem.

(b) Define :-

(i) Relatively compact.
(ii) Spectrum
(iii) Linear operator

OR

(d) Let T_n be a sequence of compact linear operator, from a normed space x into a Banach space y. If T_n is strongly operator convergent, then prove that T is compact.

(e) Show that the operator T, is defined by $T(x)=f(x)z$ is compact, where z be any fixed element of x and $f\in x$

Q.2 (a) State and prove weak convergence theorem.

(b) Let B be a subset of a metric space x. if B is totally bounded then prove that B is finite ∈- net and B is seperable.

OR

(a) Show that the operator T, is defined by $T(x)=f(x)z$ is compact, where z be any fixed element of x and $f\in x$.

(b) Let T be a bounded, self adjoint linear operators on a complex Hilbert space H and $\sigma(T)$ is the spectrum of T then prove that

$$\sigma(T) C[m, M] \; where \; m = \inf_{\|x\|=1} < T_x, x > and \; M = \sup_{\|x\|=1} < T_x, x >$$

Q.3 (a) Prove that every positive bounded self adjoint linear operator T on a complex Hilbert space H has a positive square root and is unique.

(b) If two bounded self adjoint linear operators S and T on the Hilbert space
` H are positive and commute then prove that ST is positive.

OR

(b) Let T is compact linear operator defined on the normed space X. Then prove that $N(T_n)$ is finite dimensional with $T \neq 0$

(c) Define – Relative compact metric space.

Q.4 (a) Prove that production and summation of projection is also a projection.

(c) Define projection operators. Also prove a bounded linear operator p on a Hilbert space H is a projection if P is self adjoint and idempotent.

OR

(a) Prove that B is relatively compact if and only if B is totally bounded where B is Subset of complete metric space x.

(b) Define Residual spectrum.

Q.5 (a) If two bounded self adjoint linear operators S and T on the Hilbert space
` H are positive and commute then prove that ST is positive.

(b) If T is linear operator defined on a complex Hilbert space H and also satisfied symmetric condition then prove that T is bounded.

OR

(c) Let $S: D(S) \rightarrow H$ and T:D(T) $\rightarrow H$ be linear operators. Which are densely defined in a complex Hilbert space H then prove that
If $D(T^*)$ is dense in H then $T \subset T^{**}$.

(d) Prove that B is relatively compact if and only if B is totally bounded where B is Subset of complete metric space x.

Enrollment No……………..

Master of Science (Mathematics)
Fourth Semester Main Examination, Aug-Sep 2020
Advanced Numerical Analysis-II [MSM404T]

Time: 3:00 Hrs **Max Marks 85**

Note : Attempt all questions. Question no. 1 to Q. no.5 has two parts. Part A is 10 marks and part B is 7 marks.

Q.1 (a) Explain Multistep method.

(b) Find the Jacobian Matrix for the system of equations.

$f_1(x,y) = x^2 + y^2 - x = 0$, $f_2(x,y) = x^2 - y^2 - y = 0$ at the point (1,1) with $h = R = 1$.

OR

(f) Explain Richardson's extrapolation.

(g) The second order difference method is used to solve the differential equation $\mu'' + w^2\mu = 0$. Show that the solution of the difference equation.

Q.2 (a) Explain stability of PH_P CMC method.

(c) Test the stability of second order nystrom method to the test equation $\mu' = \lambda\, \mu$.

OR

(c) Apply the second order Adanes-Bash forth method to the test equation $\mu' = \lambda\, \mu$ and hence illustrate the difference between absolute and relative stabilities.

(b) Prove that the order P of an A-stable linear multistep method cannot exceed 2 and the method must be implicit.

Q.3 (a) Explain linear second order differential equation by shooting method for boundary conditions of the second kind and third kind.

(b) Derive the general solution of linear second order differential equation by shooting method for boundary condition of first kind.

OR

(d) Use the shooting method to solve the mixed boundary value problem

$$\mu'' = \mu - 4xe^x, \quad 0 < x < 1$$
$$\mu(0) = \mu'(0) = -1 \quad \mu(1) + \mu'(1) = -e$$

Use the Taylor series method of third order to solve the initial value problems with h=0.5

(e) Use a second order method for the solution of the boundary value problem

$$\mu'' = x\mu + 1, \quad 0 < x < 1$$
$\mu(0) = \mu'(0) = 1, \quad \mu(1) = 1$ with step length h=0.25

Q.4 (a) Solve the boundary value problem

$$\mu'' = \mu + x$$
$$\mu(0) = \mu(1) = 0 \quad with \;\; h = \frac{1}{4} \;\; by\ Numerov\ method$$
$$\mu'' = x\mu + 1, \quad 0 < x < 1$$
$\mu(0) = \mu'(0) = 1, \quad \mu(1) = 1$ with step length h=0.25

(d) Solve the boundary value problem

$$\mu'' = \mu + x$$
$$\mu(0) = \mu(1) = 0 \quad with \;\; h = \frac{1}{4} \;\; by\ Numerov\ method.$$

OR

(a) Solve the boundary value problem. $\mu' = \mu' + 1$

$$\mu(0) = 1, \quad \mu(1) = 2(e-1)$$
$$with\ h = \frac{1}{3} \quad using\ second\ order\ method.$$

(b) Explain derivation boundary condition.

Q.5 (a) Explain Ritz method for boundary value problem.

(b) Solve the boundary value problem.

$\mu'' + \mu = x, \quad 0 < x < 1$

$\mu(0) = 4 \ , \quad \mu'(1) = 1$

Using the Ritz finite element method with linear piecewise polynomial for two elements of equal lengths

OR

(e) Consider the boundary value problem

$\mu'' + 2\mu = x, \quad 0 < x < 1$

$\mu(0) = 0 \ , \quad \mu(1) = 1$

Determine the coefficient of the approximate solution function $w(x) = x(1-x)(a_1 + a_2 x)$ by the Ritz method.

(f) Derive the matrix form of the variational equation of linear boundary value problem. $\frac{-d}{dx}[p(x)\mu'(x)] + q(x)\mu(x) = r(x)$
Subject to the boundary condition of first kind $\mu(a) = y_1, \ \mu(b) = y_2$ by finite element method.

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Master of Science (Mathematics)

Fourth Semester Main Examination, Aug-Sep 2020

Integral Transform-II [MSM407T]

Time: 3:00 Hrs **Max Marks 85**

Note : Attempt all questions. Question no. 1 to Q. no.5 has two parts. Part A is 10 marks and part B is 7 marks.

Q.1 (a) Evaluate $L\left\{ 1 - \frac{cos2t}{t} \right\}$

(b) Find the value of $L\left\{\frac{e^{-at} - e^{-bt}}{t}\right\}$

OR

(h) Find the Laplace transform of t^2 sinat.

(i) Define Laplace transform of unit step function.

Q.2 (a) Find the La Place transform of
$$\frac{Cos\ at - Cos\ bt}{t}$$

(d) Write the application of beam.

OR

(d) Evaluate $L\left\{\int_0^t \frac{sint}{t}\ dt\right\}$

(e) Find laplace transform of t^2 Cosat.

Q.3 (a) Find the Fourier transform of (x)_

$$F(x) = \begin{cases} Cosx\ 0 & < x < a \\ 0 & x > a \end{cases}$$

(c) Find the fourier cosine transform of

$$f(x) = e^x x > o$$

OR

(a) State and prove Inversion formula for fourier complex transform.

(b) Define fourier transform and explain the shifting property of fourier transform.

Q.4 (a) Evaluate

$$L^{-1}\left[\frac{s}{(s^2+1)(s^2+4)}\right]$$

by convolution method

(b) State and prove change of scale property for fourier transform

OR

(c) State and prove persevals indentity for fourier transform.

(d) State and prove convolution theorem.

Q.5 (a) Find laplace transform of $t^2 e^t$ sin4 t.

(c) Find Laplace transform of t sinhat.

OR

(g) Find the fourier cosine transform of
$f(x) = 5e2x + 2e5^x$

(h) Write the applications of fourier transform.